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OBJECT RECONSTRUCTION PROBLEMS IN RADIATION IMAGING WITH LIMITED ANGULAR INPUT

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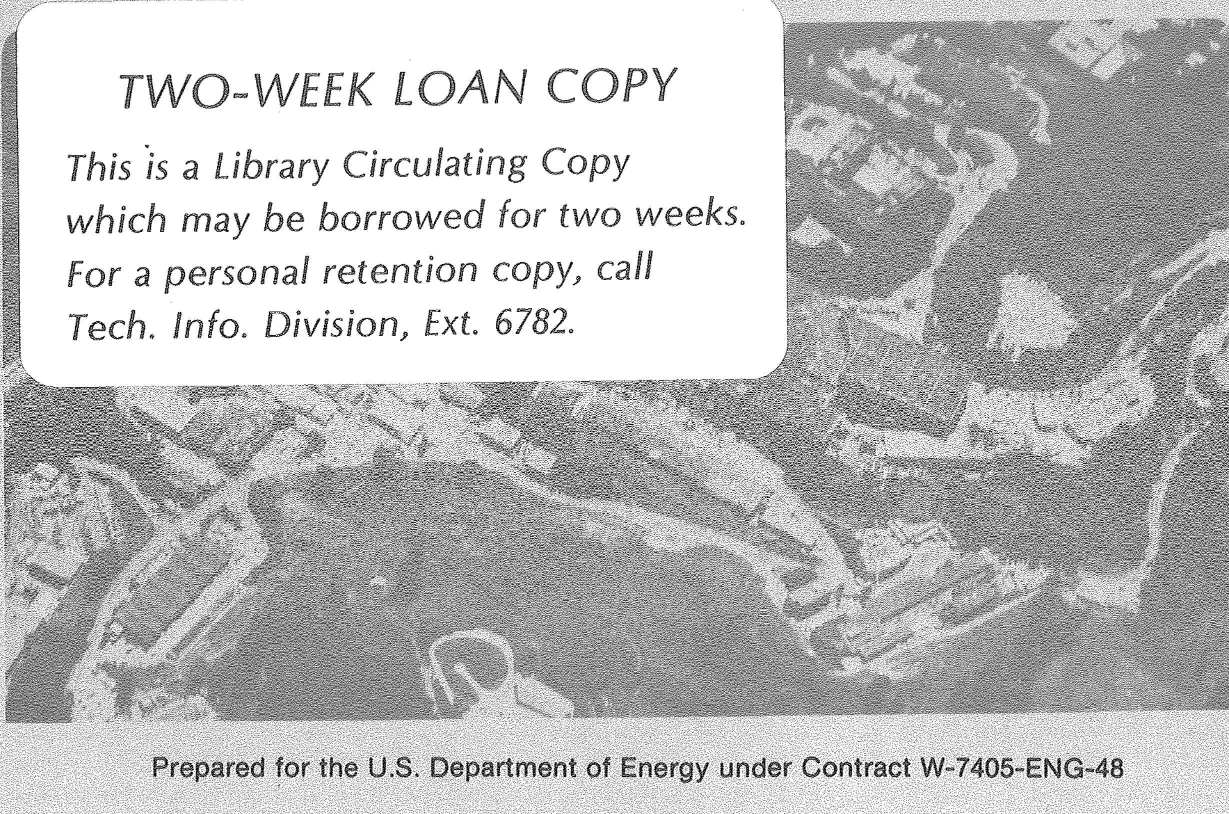

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Object Reconstruction Problems in Radiation Imaging with Limited Angular Input, K. C. TAM, and V. PEREZ-MENDEZ,* Lawrence Berkeley Laboratory, Berkeley, CA and University of California San Francisco, San Francisco, CA-Object reconstruction from limited-angle radiation imaging situations which may be encountered in computerized tomography, nuclear medicine, or electron microscopy poses computational and stability problems. We show that by using various kinds of iteration methods, one obtains an initial rapid convergence to the original object, but the final convergence is slow due to the ill-conditioned character of the eigenvalue distribution. We also show that entropy maximization methods, which have been employed in various kinds of image processing, are not expected to produce significant results in this limited-angle imaging.

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Object Reconstruction Problems in Radiation Imaging with Limited Angular Input

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Summary

In many radiation imaging applications, such as computerized tomography, nuclear medicine and electron microscopy, it may be necessary to image the object from a limited angular range. It has been shown [1,2] that in such cases the Fourier components of the object that can be measured directly are those inside a cone-shaped region in the Fourier space. The shape of this so-called "allowed cone" is determined by the space-invariant point response function of the imaging device.

In principle, all the missing-cone components of the object can be recovered if the object is known to be finite in extent. The recovering operation can be formulated as an eigenvalue problem. For each pair of specified object boundary and allowed-cone angle, there exist a complete set of eigenfunctions $\{\psi_i(\underline{k})\}$ and a corresponding set of eigenvalues $\{\lambda_i\}$, where $0 < \lambda_i < 1$ for all i. Let the Fourier components $R(\underline{k})$ of the object be represented by

$$R(\underline{k}) = \sum_i a_i \psi_i(\underline{k})$$

Then the partial knowledge of $R(\underline{k})$ in the allowed cone, say $S(\underline{k})$, can be shown to be given by

$$S(\underline{k}) = \sum_i \lambda_i a_i \psi_i(\underline{k})$$

On the surface it seems straight forward to get $R(\underline{k})$ from $S(\underline{k})$: each eigenfunction coefficient of $R(\underline{k})$ is obtained by dividing the corresponding coefficient of $S(\underline{k})$ by λ_i, which is always possible, since all eigenvalues are non-zero. In practice some of the eigenvalues are very small in magnitude, making the inversion process very unstable to noise.

An iteration algorithm has been developed in previous papers [1,2] to recover the missing-cone components by transforming the object back and forth between the object space and Fourier space. The truncation error $E_t^{(n)}(\underline{k})$ in terminating the iteration after n steps has been shown to be given by [3]:

$$E_t^{(n)}(\underline{k}) = -\sum_i a_i(1 - \lambda_i)^n \psi_i(\underline{k})$$

which goes to zero as $n \to \infty$. Since $E_t^{(n)}(\underline{k})$ does not involve any reciprocal of λ_i, the iteration procedure is stable to noise. Yet the small eigenvalues still cause trouble, since $(1 - \lambda_i)^n$ then goes to zero very slowly. A similar iteration scheme which manipulates the object back and forth between the object space and projection space has been shown to give no improvement while taking up much longer computing time [4].

Some authors [5,6] have tried reconstructing images by maximizing the entropy of the object under the constraints of the projection data. It is shown in this paper that such a procedure is not likely to produce significantly better results in limited-angle reconstruction. The reason is that the curvature of the entropy surface depends on the number of constraints that are known [7]: the larger the number of known constraints the steeper the curvature, and vice versa. The lack of knowledge in the missing cone tends to flatten the entropy surface. For typical missing-cone angles the surface is quite flat, and so a reconstructed image with entropy very close to the maximum may be very different from the true object. This fact has been demonstrated by plotting the variation of the root-mean-square error and the entropy difference between an object and the reconstructed image obtained in the Fourier transform iterations as a function of the number of iterations. The results are shown in Figure 1. It is evident that the variation of root-mean-square error is very insensitive to entropy changes.

References

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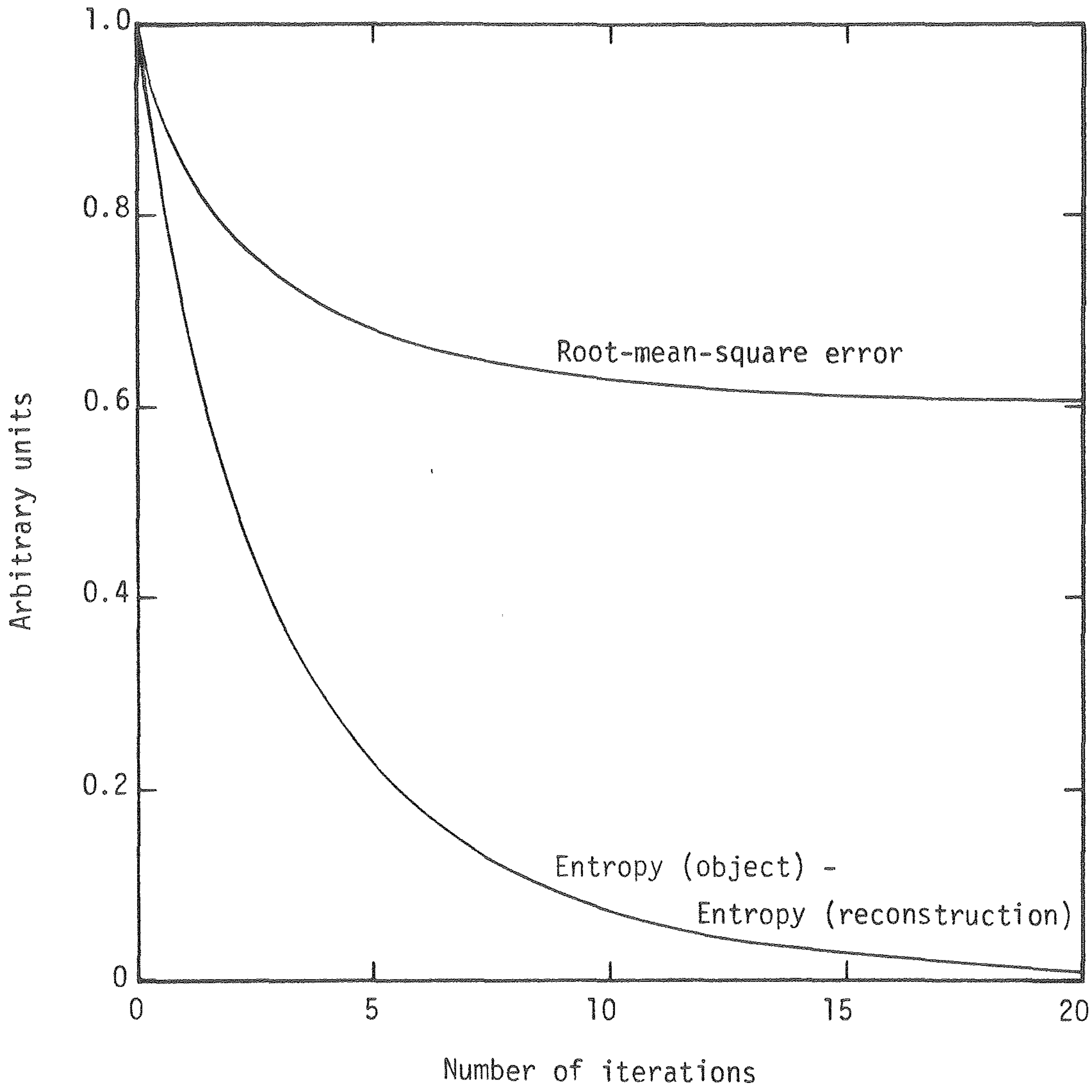

Fig. 1. The root-mean-square error and entropy difference between a 2-D object and the reconstructed image from the Fourier transform iterations plotted as a function of the number of iterations. The half-angle of the allow cone is $\tan^{-1}(0.5)$.